Delay-Dependent Stability Analysis of Multi-Area Load Frequency Control with Enhanced Accuracy and Computation Efficiency

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***Abstract*—This paper aims at improving calculation accuracy and reducing calculation burden of delay-dependent stability analysis for large-scale multi-area load frequency control (LFC) schemes in traditional and deregulated environments. The special features of the LFC models have been exploited via model reconstruction technique to equivalently transform the original LFC model into a delay-free part and a delay-related part by a transition matrix. The improvement of the calculation efficiency is obtained by decreasing both the number of decision variables and the maximal order of the linear matrix inequalities. Based on the reconstructed model, a novel augmented Lyapunov functional is constructed mainly based on the delay-related part with much lower order and its derivative is bounded with the Wirtinger inequality to establish a stability condition with less conservatism. Case studies are based on two-area LFC systems to verify the effectiveness of proposed stability criteria based on time-domain indirect method, with a great improvement of the calculation accuracy which can achieve almost accurate delay margins, like the frequency domain method. Moreover, comparing with the criterion based on the original system model without reconstruction, the calculation efficiency has been greatly improved with the cost of small deduction of accuracy.**

***Index Terms*—Load frequency control, communication delays, stability analysis, delay margin, model reconstruction**

I. Introduction

Load frequency control (LFC) has been successfully applied in power system to maintain the grid frequency and the tie-line power exchange between different control areas [1], [2]. Communication networks are employed in the LFC schemes for transmission of remote measurements to the control centre, and control signals from control centre to generator units [3]. LFC is one of the smart grid applications which utilize the communication networks in the closed-loop power system control [4]. Nowadays, power grid usually employs dedicated communication networks to implement the LFC, in which the time delay introduced in the closed-loop can be bounded below a small constant and its impact could be ignored due to the relatively slower dynamic of the LFC [5]. With the development of electricity market and smart grid technologies, open communication networks are preferred thanks to their low-cost and feasibility, for applications such as the bilateral contract between the generation and the demand sides, or the third-party LFC service integrating the distributed demand-side response. The usage of open communication networks will ultimately introduce random delays and data packet drop into the closed-loop of the LFC scheme [6], [7]. Impact of time delays should be investigated for assuring the stability of the LFC schemes.

Delay dependent stability of the LFC scheme considering the impact of communication networks in its control loop has been investigated, via frequency domain method [8], [9], [10] or time domain indirect method [11], [12]. The frequency domain method can obtain the accurate value of delay margins, but it can only deal with constant delays. To handle the time-varying and random delays, the time domain indirect methods based on Lyapunov stability theory and linear matrix inequality techniques (LMIs) have been proposed as an effective method to obtain approximate value of the delay margin [13]. The delay margins and the relationships between the delay margins and PI gains of traditional multi-area LFC scheme [11], and deregulated market environment [14], have been investigated via free-weighting matrix based stability criteria, respectively. Moreover, to improve the accuracy of delay margin obtained, less conservative stability criteria were established to ascertain delay-dependent stability of deregulated LFC systems with two additive time-varying delays [15], or stochastic interval delays [16]. The stability of the LFC scheme was analyzed by modeling load disturbances as bounded perturbations [17], and further improved via newly proposed inequalities [18], [19].

Besides of developing less conservative stability criteria to improve the calculation accuracy of delay margins, the calculation burden is another equally important performance index, especially for dealing with high-dimensional multi-area LFC problems. Zhang *et al.* has proposed the improved stability criteria with less number of the decision variables to reduce the calculation time [14]. However, the criterion is developed for generally dynamic system but not specially for the LFC problem. Many useful and special features of the LFC problem, such as the sparsity and the symmetry of the system model, have not been considered. Structure characteristics of the LFC model have been exploited via decomposing the originally developed LMI-based conditions into several lower-order

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LMIs so as to improve the calculation efficiency [20]. Note that the main work focuses on the LMI conditions obtained and the special features of the LFC model have not been utilised for deriving the stability criteria. In [21], the calculation time has been reduced by re-ranking the positions of time-delayed variables and normal state variables to decompose the system model into a delay-free part combined with a delay-related part, and then constructing the Lyapunov functional based on the relatively low-order delay-related part.

This paper further investigates the delay-dependent stability of large-scale multi-area traditional and deregulated LFC schemes, aiming at improving the calculation accuracy and the computation efficiency simultaneously. The contributions are two folds. Firstly, to increase the calculation efficiency, the special features of the LFC model are fully exploited via model reconstruction technique. The original model of the LFC problem is equivalently represented with a delay-free part and a delay-related part via applying a transition matrix. The method proposed in [21] has been extended to obtain these delay-related states just including the system states directly embedded delays, which enables to deal with LFC schemes equipped with PID type controllers. Moreover, a new stability criterion is developed mainly based on the delay-related part of the reconstructed system model. The improvement of the calculation efficiency is obtained by decreasing both the number of decision variables (NDVs) and the maximal order of the LMIs (MoLs), which differs from [20] just reducing the MoLs in order to speed up the calculation. Secondly, based on the delay-related states in the reconstructed system model, a new augmented Lyapunov functional is constructed and its derivative is estimated via a Wirtinger inequality. The augmented part in the Lyapunov functional introduces the integral terms containing the time delays in the adjacent regions of power system, which can reduce the conservatism of the time-domain indirect method and try to obtain almost accurate delay margins like the frequency domain method. Comparing with the stability criterion based on the original model, the proposed new stability criterion can obtain a great improvement of calculation efficiency, but only with a small cost of accuracy deduction.

The remaining parts of this paper are organized as follows. Section II gives the original model and the reconstructed model for traditional and deregulated multi-area LFC schemes respectively firstly, and then describes the model reconstruction technique. Section III proposes the improved stability criteria, based on the original model and the reconstructed model. In Section IV, case studies are based on two-area traditional and deregulated LFC schemes to verify the effectiveness of the stability criteria. Finally, conclusions and future work are presented in Section V.

II. MODELS OF MULTI-AREA LFC SCHEMES

The dynamic models of the traditional and deregulated LFC schemes are recalled at first. Then a model reconstruction technique is developed to transform the original model into a coupled system which consists of a delay-related subsystem and a delay-free subsystem.

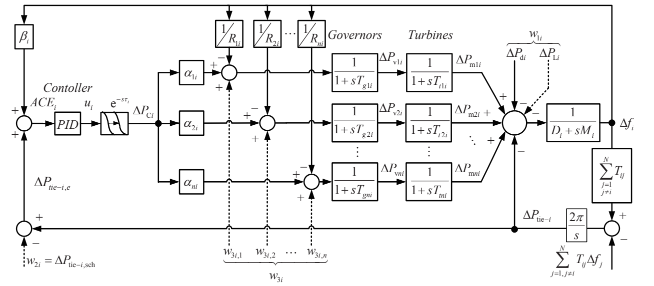

Fig. 1. LFC structure of control area (Traditional LFC: without dotted line connection; Deregulated LFC: with dotted line connection).

A. Original dynamic model

Model given in [14] has been recalled in this section, in which each area of the LFC schemes is assumed to involve n generation companies (Gencos) installed with non-reheat turbines, m distribution companies (Discos), and a PID-type LFC controller. The structure of area i is shown in Fig. 1, in which the exponential block $e^{-s\tau_i}$ shows the delays arising in the communication channels; $\Delta f_i, \Delta P_{tie-i,e}, \Delta P_{mni}$, ΔP_{vni} are the deviation of frequency, tie-line power exchange, mechanical output of generator, and valve position, respectively; $M_i, D_i, T_{gni}, T_{tni}$, R_{ni} are the moment of inertia of generator unit, generator unit damping coefficient, time constant of the governor, time constant of the turbine and speed drop, respectively; and β_i, α_{ni} are the frequency bias factor of area i, ramp rate factor, respectively.

Give area control error of area i as

$$ACE_i = \beta_i \Delta f_i + \Delta P_{tie-i,e}, \tag{1}$$

and design a PID-type LFC controller as

$$u_i(t) = -K_{Pi} ACE_i - K_{Ii} \int ACE_i - K_{Di} \frac{dACE_i}{dt} \tag{2}$$

where K_{Pi}, K_{Ii} and K_{Di} are proportional, integral and differential gain, respectively. The closed-loop models of the LFC schemes in deregulated environment can be obtained as:

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{N} A_{\mathrm{d}i} x(t-\tau_i) + B_{\mathrm{w}} \omega \tag{3}$$

where

$$x = [\bar{x}_1^T, \bar{x}_2^T, \cdots, \bar{x}_n^T]^T, \quad \bar{x}_i = [\hat{x}_i^T, \int y_i^T]^T, \quad y_i = ACE_i$$

$$\hat{x}_i = [\Delta f_i, \Delta P_{tie-i,e}, \Delta P_{m1i}, \cdots, \Delta P_{mni}, \Delta P_{v1i}, \cdots, \Delta P_{vni}]^T$$

$$A = \begin{bmatrix} \bar{A}_{11} & \cdots & \bar{A}_{1N} \\ \vdots & \ddots & \vdots \\ \bar{A}_{N1} & \cdots & \bar{A}_{NN} \end{bmatrix}, A_{\mathrm{d}i} = \begin{bmatrix} 0_{(i-1)(2n+2)\times N(2n+2)} \\ \bar{A}_{\mathrm{d}i1} \ \bar{A}_{\mathrm{d}i2} \cdots \bar{A}_{\mathrm{d}iN} \\ 0_{(N-i)(2n+2)\times N(2n+2)} \end{bmatrix}$$

$$\bar{B}_{\mathrm{w}} = \mathrm{diag}\{\bar{B}_{\mathrm{w}1}, \cdots, \bar{B}_{\mathrm{w}N}\}, \ \omega = \mathrm{diag}\{\omega_1, \cdots, \omega_N\}$$

$$\bar{A}_{\mathrm{d}ii} = -\bar{B}_i K_i \bar{C}_i, \ \bar{A}_{\mathrm{d}ij} = -\bar{B}_i K_i \bar{C}_{ij}, \ \bar{B}_{\mathrm{w}i} = \bar{F}_i - \bar{B}_i K_i \bar{D}_i$$

$$\bar{A}_{ii} = \begin{bmatrix} A_i & 0 \\ C_i & 0 \end{bmatrix}, \ \bar{A}_{ij} = \begin{bmatrix} A_{ij} & 0 \\ 0 & 0 \end{bmatrix}, \ \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \ \bar{F}_i = \begin{bmatrix} F_i \\ D_i \end{bmatrix}$$

$$\bar{C}_i = \begin{bmatrix} C_i & 0 \\ 0 & 1 \\ C_i A_i & 0 \end{bmatrix}, \bar{C}_{ij} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ C_i A_{ij} & 0 \end{bmatrix}, \bar{D}_i = \begin{bmatrix} D_i \\ 0 \\ C_i F_i \end{bmatrix}$$

$$A_i = \begin{bmatrix} A_{11i} & A_{12i} & 0_{2\times n} \\ 0_{n\times 2} & A_{22i} & A_{23i} \\ A_{31i} & 0_{n\times n} & A_{33i} \end{bmatrix}, A_{ij} = \begin{bmatrix} 0 & 0 & 0_{1\times 2n} \\ 2\pi T_{ij} & 0 & 0_{1\times 2n} \\ 0_{2n\times 1} & 0_{2n\times 1} & 0_{2n\times 2n} \end{bmatrix}$$

$$A_{11i}=\begin{bmatrix}-\frac{D_i}{M_i} & -\frac{1}{M_i}\\ 2\pi\sum_{j=1,j\neq i}^{N}T_{ij} & 0\end{bmatrix}, A_{12i}=\begin{bmatrix}\frac{1}{M_i} & \cdots & \frac{1}{M_i}\\ 0 & \cdots & 0\end{bmatrix}$$

$$A_{22i}=-A_{23i}=-\mathrm{diag}\left\{\frac{1}{T_{t1i}},\cdots,\frac{1}{T_{tni}}\right\}$$

$$A_{31i}=-\begin{bmatrix}\frac{1}{R_{1i}T_{t1i}} & \cdots & \frac{1}{R_{ni}T_{tni}}\\ 0 & \cdots & 0\end{bmatrix}^T$$

$$A_{33i}=-\mathrm{diag}\left\{\frac{1}{T_{g1i}},\cdots,\frac{1}{T_{gni}}\right\}$$

$$B_i=\begin{bmatrix}0_{2\times1}\\ 0_{n\times1}\\ B_{3i}\end{bmatrix}, B_{3i}=\begin{bmatrix}\frac{\alpha_{1i}}{T_{g1i}},\cdots,\frac{\alpha_{ni}}{T_{gni}}\end{bmatrix}^T, F_i=\begin{bmatrix}-\frac{1}{M_i}\\ 0_{(2n+1)\times1}\end{bmatrix}$$

$$C_i=[\beta_i,1,0_{1\times2n}], \beta_i=\sum_{j=1}^{n}\frac{1}{R_{ji}}+D_i$$

The model of traditional LFC scheme can be obtained by omitting the dotted line included part in Fig. 1, and redefining the F_i and D_i of B_{w} in (3) as $F_i=\begin{bmatrix}\frac{-1}{M_i} & 0 & 0\\ 0 & -2\pi & 0\end{bmatrix}^T, D_i=[0,\ 0]$. Detailed procedure of obtaining above models can refer to [14], which is omitted here due to page limitation.

As mentioned in [24], external disturbances do not affect the internal stability of LFC schemes. Thus, the stability analysis of LFC schemes can be conducted under the following disturbance-free model

$$\dot{x}(t)=\sum_{i=0}^{N}A_ix(t-\tau_i) \tag{4}$$

where $A_0=A_m$ and $A_i\in\{A_{md1},A_{md2},\cdots,A_{mdN}\}$ are obtained by reordering the time delays in (3) as follows

$$0=\tau_0\leq\tau_1\leq\cdots\leq\tau_N. \tag{5}$$

B. Reconstructed model

Most of stability criteria developed in literature for (4) are obtained directly based on the whole system model [17] and the high-dimensional model of the multi-area LFC schemes makes the LMI-based conditions with many decision variables and very high-order LMIs. In practice, it often takes unacceptable calculation time to apply those criteria for finding a solution to multi-area LFC problems. In fact, those factors (NDVs and MoLs) related with the calculation time are strongly dependent on the delay-related states of the whole system and the LFC problem only has few delay- related states. It can be shown in Section III.A that less delay-related states used during the construction of the Lyapunov functional make it much easier to check the LMI-based conditions.

Thus, a model reconstruction technique that separates the delay-related states from delay-free states is developed in this paper by analyzing the characteristic of system (4).

Fig. 1 shows that time delays are introduced by

$$\Delta P_{Ci}(t)=u_i(t-\tau_i) \tag{6}$$

which results in only two delay-related states, Δf_i and ΔP_{tie-i}. Moreover, those two states satisfy the following relationships:

$$\Delta\dot{P}_{vki}(t)=f(\Delta f_i(t-\tau_i),\Delta P_{tie-i}(t-\tau_i)) \tag{7}$$

$$\Delta\dot{f}_i(t)=g_1(x_i(t)),\quad \Delta\dot{P}_{tie-i}(t)=g_2(x_i(t)) \tag{8}$$

where $f(\cdot)$ and $g_i(\cdot)$ are suitable functions. That is, the closed-loop LFC schemes only have several delay-related states. Hence, system (4) could be rewritten as

$$\begin{cases}\dot{x}_1(t)=A_{11}x_1(t)+A_{12}x_2(t)\\ \dot{x}_2(t)=A_{21}x_1(t)+A_{22}x_2(t)+\sum_{i=1}^{N}A_{di}x_1(t-\tau_i)\end{cases} \tag{9}$$

where $x_1\in R^{n_1}$ is the combination of these delayed states, $x_2\in R^{n_2}$ donates other states. System (9) is equivalent to

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix}=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\sum_{i=1}^{N}\begin{bmatrix}0 & 0\\ A_{di} & 0\end{bmatrix}\begin{bmatrix}x_1(t-\tau_i)\\ x_2(t-\tau_i)\end{bmatrix}. \tag{10}$$

In fact, transformed model (10) can be obtained from original model (4) via a nonsingular transformation, $\begin{bmatrix}x_1\\ x_2\end{bmatrix}=Tx$ (T is a nonsingular $n\times n$ matrix). Procedure of the transformation is given as follows to show how to obtain system matrices of (10):

Step1. Find out the delay-related state vector x_1. Find out the column numbers of non-zero column of matrix $\mathcal{A}_d=\sum_{i=1}^{N}A_{di}$, which are the same to the row numbers of delayed states in $x(t)$ based on system (4), denote those numbers by $a_k, a_k\in\{1,2,\cdots,n_1+n_2\}$, and denote the other column numbers of matrix $\mathcal{A}_d$ by $b_k, b_k\in\{1,2,\cdots,n_1+n_2\}, b_k\neq a_k$;

Step2. Define the nonsingular matrix T. Based on $\begin{bmatrix}x_1\\ x_2\end{bmatrix}=Tx$, the role of matrix T is to reorder the state vector $x(t)$ by moving delayed states into begining part. Thus, matrix T can be given as

$$T=[E_{a_1},E_{a_2},\cdots,E_{a_{n_1}},E_{b_1},E_{b_2},\cdots,E_{b_{n_2}}]^T$$

where $E_k=[0_{1\times(k-1)},1,0_{1\times(n_1+n_2-k)}]^T$.

Step3. Calculate system matrices of system (10). Transforming system (4) with $\tilde{x}=\begin{bmatrix}x_1\\ x_2\end{bmatrix}=Tx$ yields

$$\dot{\tilde{x}}(t)=TA_0T^{-1}\tilde{x}(t)+\sum_{i=1}^{N}TA_iT^{-1}\tilde{x}(t-\tau_i).$$

That is,

$$\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}=TA_0T^{-1}, \begin{bmatrix}0 & 0\\ A_{di} & 0\end{bmatrix}=TA_iT^{-1}. \tag{11}$$

A simple example is given to illustrate the above procedure. The related system matrices for a two-area deregulated LFC scheme equipped with PI controller are given as [14]:

$$A_1 = \begin{bmatrix} 0_{4\times1} & 0_{4\times1} & 0_{4\times4} & 0_{4\times1} & 0_{4\times6} \\ -0.7083 & -1.6667 & 0_{1\times4} & -1.6667 & 0_{1\times6} \\ -0.5313 & -1.2500 & 0_{1\times4} & -1.2500 & 0_{1\times6} \\ 0_{7\times1} & 0_{7\times1} & 0_{7\times4} & 0_{7\times1} & 0_{7\times6} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0_{10\times1} & 0_{10\times1} & 0_{10\times5} & 0_{10\times1} & 0_{10\times4} & 0_{10\times1} \\ 0 & 1.6667 & 0_{1\times5} & -0.6610 & 0_{1\times4} & -1.6667 \\ 0 & 1.4286 & 0_{1\times5} & -0.5666 & 0_{1\times4} & -1.4286 \\ 0 & 0 & 0_{1\times5} & 0 & 0_{1\times4} & 0 \end{bmatrix}.$$

It can be found that $a_k \in \{1, 2, 7, 8, 13\}$, $n_1 = 5$, and $n_1 + n_2 = 13$. Thus, $T = [E_1, E_2, E_7, E_8, E_{13}, E_3, E_4, E_5, E_6, E_9, E_{10}, E_{11}, E_{12}]^T$. Therefore, it follows (11) that

$$A_{d1} = \begin{bmatrix} 0_{4\times1} & 0_{4\times1} & 0_{4\times1} & 0_{4\times2} \\ -0.7083 & -1.6667 & -1.6667 & 0_{1\times2} \\ -0.5313 & -1.2500 & -1.2500 & 0_{1\times2} \\ 0_{2\times1} & 0_{2\times1} & 0_{2\times1} & 0_{2\times2} \end{bmatrix}$$

and $A_{ij}, i = 1, 2; j = 1, 2, A_{d2}$ are omitted here.

Remark 1. *It will be shown that, in next section, the smaller n_1 (the number of delayed states) is, the simpler the obtained stability criterion is. From this point of view, the advantage of the proposed model reconstruction method compared with the one used in [21] can be found. By recalling the method of [21], original system (4) will be transformed as*

$$\begin{bmatrix} \dot{\check{x}}_1(t) \\ \dot{\check{x}}_2(t) \end{bmatrix} = \begin{bmatrix} \check{A}_{11} & \check{A}_{12} \\ \check{A}_{21} & \check{A}_{22} \end{bmatrix} \begin{bmatrix} \check{x}_1(t) \\ \check{x}_2(t) \end{bmatrix} + \sum_{i=1}^{N} \begin{bmatrix} 0 & 0 \\ 0 & \check{A}_{di} \end{bmatrix} \left[\check{x}_1(t - \tau_i) \right] \quad (12)$$

It can be found that the vector $\check{x}_2$ consists of the states $\check{x}_{21} \in R^{m_1}$, whose themselves contain delays and m_1 depends on the non-zero column of matrix $\mathcal{A}_d$, and the states $\check{x}_{22} \in R^{m_2}$, whose derivatives are related to delayed states and m_2 depends on the non-zero row of matrix $\mathcal{A}_d$. For the above example, the column numbers of non-zero column of matrix $\mathcal{A}_d$ are $\Omega_1 = \{1, 2, 7, 8, 13\}$, and the row numbers of non-zero row of matrix $\mathcal{A}_d$ are $\Omega_2 = \{5, 6, 11, 12\}$. Then, $\check{x}_2$ contains 9 states, including all the k-th state of $x(t)$, $k \in \Omega_1 \cup \Omega_2 = \{1, 2, 5, 6, 7, 8, 11, 12, 13\}$. Therefore, the proposed reconstructed method is less strict than the one used in [21], as only the system states directly embedded with delays are included in the delay-related part, while in [21] some states indirectly linked with the delayed states are also included in the delay-related part.

III. New Stability Criterion with Improved Accuracy and Efficiency

This section presents a time-domain indirect method based on the Lyapunov stability to achieve the almost accurate delay margins with improved calculation efficiency.

A. Improved stability criteria

In order to illustrate how the number of delay-related states influences the calculation efficiency and also to easily show what the advantages of the proposed criterion are, a stability criterion based on the original system (4) is developed via constructing a newly augmented Lyapunov functional and estimating its derivative by the Wirtinger inequality [22], which is firstly given as follows.

Lemma 1. *For given scalars $\tau_i, i = 0, 1, ..., N$ satisfied with (5), system (4) is globally asymptotically stable, if there exist symmetric $(N+1)(n_1+n_2) \times (N+1)(n_1+n_2)$ matrix $P > 0$, symmetric $(n_1 + n_2) \times (n_1 + n_2)$ matrices $Q_i > 0, R_i > 0, i = 1, 2, ..., N$, such that the following LMI holds*

$$\Pi = \Xi + \Xi^T + \sum_{i=1}^{N} \Psi_i < 0 \quad (13)$$

where

$$\Xi = \begin{bmatrix} e_1 \\ (\tau_1 - \tau_0) e_{N+2} \\ \vdots \\ (\tau_N - \tau_{N-1}) e_{2N+1} \end{bmatrix}^T P \begin{bmatrix} e_s \\ e_1 - e_2 \\ \vdots \\ e_N - e_{N+1} \end{bmatrix}$$

$$\Psi_i = e_i^T Q_i e_i - e_{i+1}^T Q_i e_{i+1} + (\tau_i - \tau_{i-1})^2 e_s^T R_i e_s - \begin{bmatrix} e_i - e_{i+1} \\ e_i + e_{i+1} - 2e_{N+1+i} \end{bmatrix}^T \begin{bmatrix} R_i & 0 \\ 0 & 3R_i \end{bmatrix} \begin{bmatrix} e_i - e_{i+1} \\ e_i + e_{i+1} - 2e_{N+1+i} \end{bmatrix}$$

$$e_s = \sum_{j=0}^{N} A_j e_{j+1}$$

$$e_i = \left[0_{n\times(i-1)n}, I_{n\times n}, 0_{n\times(2N+1-i)n} \right], i = 1, 2, ..., 2N + 1.$$

Proof. For original system (4), the above criterion can be easily obtained by following the common procedure used for analyzing time-delay systems [25]. Specifically, Constructing the following Lyapunov functional candidate

$$V(t) = \begin{bmatrix} x(t) \\ \int_{t-\tau_1}^{t-\tau_0} x(s)ds \\ \vdots \\ \int_{t-\tau_N}^{t-\tau_{N-1}} x(s)ds \end{bmatrix}^T P \begin{bmatrix} x(t) \\ \int_{t-\tau_1}^{t-\tau_0} x(s)ds \\ \vdots \\ \int_{t-\tau_N}^{t-\tau_{N-1}} x(s)ds \end{bmatrix} + \sum_{i=1}^{N} \int_{t-\tau_i}^{t-\tau_{i-1}} x^T(s) Q_i x(s) ds + \sum_{i=1}^{N} (\tau_i - \tau_{i-1}) \int_{-\tau_i}^{-\tau_{i-1}} \int_{t+\theta}^{t} \dot{x}^T(s) R_i \dot{x}(s) ds d\theta. \quad (14)$$

with $P \in R^{(N+1)(n_1+n_2)\times(N+1)(n_1+n_2)}$, $Q_i \in R^{(n_1+n_2)\times(n_1+n_2)}$, and $R_i \in R^{(n_1+n_2)\times(n_1+n_2)}$ being matrices to be determined, and using Wirtinger inequality to estimate the following term appearing in $\dot{V}(t)$,

$$-\int_{t-\tau_i}^{t-\tau_{i-1}} \dot{x}^T(s) R_i \dot{x}(s) ds \quad (15)$$

yields

$$\dot{V}(t) \le \xi_1^T(t) \Pi \xi_1(t) \quad (16)$$

where $\xi_1^T(t) = \Big[x^T(t), x^T(t - \tau_1), ..., x^T(t - \tau_N), \int_{t-\tau_1}^{t-\tau_0} \frac{x^T(s)}{\tau_1 - \tau_0} ds, \cdots, \int_{t-\tau_N}^{t-\tau_{N-1}} \frac{x^T(s)}{\tau_N - \tau_{N-1}} ds \Big]$. It is obvious that the holding of LMI-based condition in Lemma 1 leads to $V(t) > 0$ and $\dot{V}(t) \le -\varepsilon \|x(t)\|^2$ for a sufficient small scalar $\varepsilon > 0$, which shows the asymptotical stability of system (4).

It can be seen from LKF (14) that all Q_i- and R_i-dependent terms are given to deal with the delayed states, and those terms introduce many decision variables (the scalars in Q_i- and R_i to be determined) to the LMIs of Lemma 1. Moreover, it can be found in (16) that many delayed states related information including in $\xi_1(t)$ increase the order of matrix Π, which in turn leads to the high-dimensional of the LMIs. That is, two key calculation time related factors, the NDVs and the MoLs, are strongly linked to the delayed states. Therefore, the number of delayed states is important for simplifying the LMIs to improve the calculation efficiency, and the smaller such number is, the simpler the LMIs are. Obviously, it is not an optimal treatment to consider all states as delayed states like original system (4) does.

According to the reconstructed system model (10), the following improved stability criterion is developed by constructing a Lyapunov functional based on the delay-related part.

Theorem 1. *For given scalars $\tau_i, i = 0, 1, ..., N$ satisfied with (5), system (9) is globally asymptotically stable, if there exist symmetric $((N+1)n_1+n_2)\times((N+1)n_1+n_2)$ matrix $P_1 > 0$, symmetric $n_1 \times n_1$ matrices $U_i > 0, Z_i > 0, i = 1, 2, ..., N$, such that the following LMI holds*

$$\bar{\Pi} = \Phi + \Phi^T + \sum_{i=1}^{N} \Theta_i < 0 \tag{17}$$

where

$$\begin{aligned}
\Phi &= \begin{bmatrix} \bar{e}_1 \\ \bar{e}_0 \\ (\tau_1 - \tau_0)\bar{e}_{N+2} \\ \vdots \\ (\tau_N - \tau_{N-1})\bar{e}_{2N+1} \end{bmatrix}^T P_1 \begin{bmatrix} \bar{e}_{s1} \\ \bar{e}_{s0} \\ \bar{e}_1 - \bar{e}_2 \\ \vdots \\ \bar{e}_N - \bar{e}_{N+1} \end{bmatrix} \\
\Theta_i &= \bar{e}_i^T U_i \bar{e}_i - \bar{e}_{i+1}^T U_i \bar{e}_{i+1} + (\tau_i - \tau_{i-1})^2 \bar{e}_{s1}^T Z_i \bar{e}_{s1} \\
&\quad - \begin{bmatrix} \bar{e}_i - \bar{e}_{i+1} \\ \bar{e}_i + \bar{e}_{i+1} - 2\bar{e}_{N+1+i} \end{bmatrix}^T \begin{bmatrix} Z_i & 0 \\ 0 & 3Z_i \end{bmatrix} \begin{bmatrix} \bar{e}_i - \bar{e}_{i+1} \\ \bar{e}_i + \bar{e}_{i+1} - 2\bar{e}_{N+1+i} \end{bmatrix} \\
\bar{e}_{s1} &= A_{11}\bar{e}_1 + A_{12}\bar{e}_0 \\
\bar{e}_{s0} &= A_{21}\bar{e}_1 + A_{22}\bar{e}_0 + \sum_{i=1}^{N} A_{di}\bar{e}_{i+1} \\
\bar{e}_i &= \left[0_{n_1\times(i-1)n_1}, I_{n_1\times n_1}, 0_{n_1\times(2N+1-i)n_1}, 0_{n_1\times n_2}\right], \\
&\quad i = 1, 2, ..., 2N+1 \\
\bar{e}_0 &= \left[0_{n_2\times(2N+1)n_1}, I_{n_2\times n_2}\right].
\end{aligned}$$

Proof. The above criterion can be directly obtained by replacing the Lyapunov functional (14) with the following one:

$$\begin{aligned}
\tilde{V}(t) &= \begin{bmatrix} x_1(t) \\ x_2(t) \\ \int_{t-\tau_1}^{t-\tau_0} x_1(s)ds \\ \vdots \\ \int_{t-\tau_N}^{t-\tau_{N-1}} x_1(s)ds \end{bmatrix}^T P_1 \begin{bmatrix} x_1(t) \\ x_2(t) \\ \int_{t-\tau_1}^{t-\tau_0} x_1(s)ds \\ \vdots \\ \int_{t-\tau_N}^{t-\tau_{N-1}} x_1(s)ds \end{bmatrix} \\
&\quad + \sum_{i=1}^{N} \int_{t-\tau_i}^{t-\tau_{i-1}} x_1^T(s) U_i x_1(s) ds \\
&\quad + \sum_{i=1}^{N} (\tau_i - \tau_{i-1}) \int_{-\tau_i}^{-\tau_{i-1}} \int_{t+\theta}^{t} \dot{x_1}^T(s) Z_i \dot{x_1}(s) ds d\theta.
\end{aligned} \tag{18}$$

with $P_1 \in R^{((N+1)n_1+n_2)\times((N+1)n_1+n_2)}$, $U_i \in R^{n_1\times n_1}$, and $Z_i \in R^{n_1\times n_1}$ being matrices to be determined. Then, based on the LMIs in Theorem 1, using Wirtinger inequality to estimate the integral term appearing in $\dot{\tilde{V}}(t)$ yields $\dot{\tilde{V}}(t) \le \xi_2^T(t)\bar{\Pi}\xi_2(t) \le -\varepsilon \|x(t)\|^2$ with $\xi_2^T(t) = \Big[x_1^T(t), x_1^T(t-\tau_1), \cdots, x_1^T(t-\tau_N), \int_{t-\tau_1}^{t-\tau_0} \frac{x_1^T(s)}{\tau_1-\tau_0} ds, \cdots, \int_{t-\tau_N}^{t-\tau_{N-1}} \frac{x_1^T(s)}{\tau_N-\tau_{N-1}} ds, x_2^T(t)\Big]$, which guarantees the asymptotical stability of system (9).

The advantages of Theorem 1 compared with Lemma 1 can be found from the NDVs and the MoLs, which are respectively given as follows

$$\begin{aligned}
n_L &= \frac{(N+1)(n_1+n_2)((N+1)(n_1+n_2)+1)}{2} \\
&\quad + \frac{2N(n_1+n_2)((n_1+n_2)+1)}{2}
\end{aligned} \tag{19}$$

$$n_T = \frac{((N+1)n_1+n_2)((N+1)n_1+n_2+1)+2Nn_1(n_1+1)}{2} \tag{20}$$

$$m_L = (2N+1)(n_1+n_2) \tag{21}$$

$$m_T = (2N+1)n_1 + n_2 \tag{22}$$

where n_L, n_T, m_L, and m_T respectively indicate the the NDVs and the MoLs of Lemma 1 and Theorem 1.

As discussed in Section II.A, only few delayed states exist in system (10), i.e., $n_1 \ll n_1 + n_2$, which implies $n_T \ll n_L$. Hence, the computational burden subjected to the high-dimensional system can be greatly released. Moreover, compared with Jenson inequality used in [14], the tighter Wirtinger inequality is used in this paper such that the Theorem 1 is less conservative than that in [14].

B. Summary of analysis steps

The steps of delay margin calculation can be briefly summarized as follows.

Step1. Model establishment. The original dynamic model of the closed-loop LFC schemes is presented in Section II.A and then a reconstructed model is proposed, according to the method shown in Section II.B.

Step2. Stability criteria development. Based on the original model, the stability condition is established via constructing a new LKF and bounding its derivative tightly. The second theorem is developed under the reconstructed model.

Step3. Calculation accuracy verification. The delay margins are calculated based on the developed stability criteria by using the MATLAB/LMI toolbox and following the algorithm given in [14]. Carry out the simulation tests to show the proposed stability criteria can obtain almost accurate delay margins.

Step4. Calculation efficiency improvement. Comparing with the stability criterion based on the original model, the improvement of calculation efficiency is verified through the comparison of NDVs, MoLs and calculation time.

IV. Case studies

Case studies based on two-area LFC schemes in both traditional and deregulated environments are carried out to show the advantages of the proposed criterion. The same parameters of the LFC schemes as [14] can be found in Appendix I. The criterion based on the original model without reconstruction is used to demonstrate the improvement of calculation efficiency in terms of NDVs, MoLs and calculation time.

A. Accuracy verification

In order to demonstrate the accuracy of the delay margins computed via this paper, the simulation tests based on Matlab/Simulink platform are carried out to calculate the real value of delay margins. The proposed criteria can provide nearly accurate delay margins, compared with other Lyapunov based indirect methods always with some approximations like [14] and [21], which is verified by making comparisons with the actual delay margins (obtained by simulation).

For the traditional two-area LFC schemes with PI controllers ($K_P = 0.4, K_I = 0.2$) or PID controllers ($K_P = 0.2, K_I = 0.2, K_D = 0.2$), the delay margins calculated via the proposed Theorem 1, together with the ones provided by [14], [21], and Lemma 1, are listed in Tables I and II respectively, in which $\tau = \sqrt{\tau_1^2 + \tau_2^2}$ represents magnitude of τ_1 and τ_2 and $\theta = \tan^{-1}(\tau_1/\tau_2)$. The boundaries of stability regions based on delay margins are shown in Figs. 3 and 4 respectively. Similarly, the delay margins of the deregulated two-area LFC scheme with PI controllers ($K_P = 0.1, K_I = 0.2$) or PID controllers ($K_P = 0.05, K_I = 0.2, K_D = 0.04$), calculated by different stability criteria, are listed in Tables III and IV respectively, and the corresponding stability regions are shown in Figs. 5 and 6 respectively.

For the two-area traditional LFC scheme equipped with PI controllers or PID controllers, provided that the generator rate constraint is ± 0.1 pu/min and a step load disturbance with 0.1 pu amplitude appears at $t = 10s$ in two areas ($\Delta P_{di} = 0.1$pu), the simulated values are shown in Tables I and II. Similarly, for deregulated case, the contract between the Discos and the Gencos is given as following AGPM:

$$AGPM = \begin{bmatrix} 0.5 & 0.25 & 0 & 0.3 \\ 0.2 & 0.25 & 0 & 0 \\ 0 & 0.25 & 1 & 0.7 \\ 0.3 & 0.25 & 0 & 0 \end{bmatrix} \tag{23}$$

A step load disturbance of 0.1 pu amplitude is demanded at $t = 20s$ by each Disco in two areas ($\Delta P_{Li} = 0.4$pu) , and Disco 1 in area 1 and Disco 2 in area 2 all demand 0.08 pu as un-contracted loads ($\Delta P_{di} = 0.16$pu). The obtained simulation values are also listed in Tables III and IV.

Particularly, for the two-area deregulated LFC scheme equipped with a PI controller ($K_P = 0.1, K_I = 0.2$), the frequency deviation of Area 1 with different time delays is shown in Fig. 2. It is revealed that the LFC scheme is marginally stable with $\tau = 11.48s$, which is closely compatible with the calculated delay margin (11.45s shown in Table III). Hence, the high accuracy of the proposed stability condition is demonstrated.

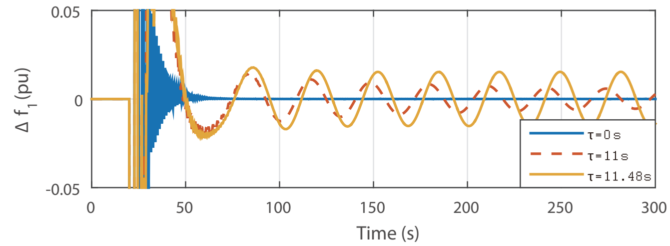

Fig. 2. Frequency deviation area 1 of two-area deregulated LFC scheme with different delays.

Meanwhile, the error values between delay margins calculated via different theoretical methods and actual value coming from simulations are also included in Tables I-IV.

Results given in Tables I-IV and Figs. 2-6 show the great improvement of the calculation accuracy. Table I shows that results obtained by Theorem 1 proposed in this paper can achieve much smaller error than stability criteria developed in paper [21] and [14]. For example, when $\theta = \{0°, 20°, 70°, 90°\}$, the errors have been reduced from $47\% - 33\%$ to $18\% - 3\%$. Moreover, almost accurate results can be obtained in Table I for $\theta = \{40°, 45°, 50°\}$, and for all results in Table II to IV, with error less than 1.5%. Thus, it is fair to conclude that the delay margins obtained by Theorem 1 can almost approach to the real value of delay margin (obtained by simulation).

TABLE I
Delay margins of traditional two-area LFC scheme with PI controller ($K_P = 0.4, K_I = 0.2$)

	τ(s)				
θ	[21] (error)	[14] (error)	Lem.1 (error)	The.1 (error)	Real Value
0°	4.89 (42.74%)	5.36 (37.24%)	7.53 (11.83%)	7.13 (16.51%)	8.54
20°	4.93 (45.76%)	5.96 (34.43%)	8.73 (3.96%)	8.15 (10.34%)	9.09
40°	5.97 (46.46%)	7.18 (35.61%)	11.11 (0.36%)	11.10 (0.45%)	11.15
45°	6.17 (48.37%)	7.55 (36.82%)	11.87 (0.67%)	11.84 (0.92%)	11.95
50°	5.71 (48.28%)	7.19 (34.87%)	10.97 (0.63%)	10.97 (0.63%)	11.04
70°	4.78 (46.89%)	5.97 (33.67%)	8.65 (3.89%)	7.86 (12.67%)	9.00
90°	4.86 (42.69%)	5.37 (36.67%)	7.59 (10.50%)	6.98 (17.69%)	8.48

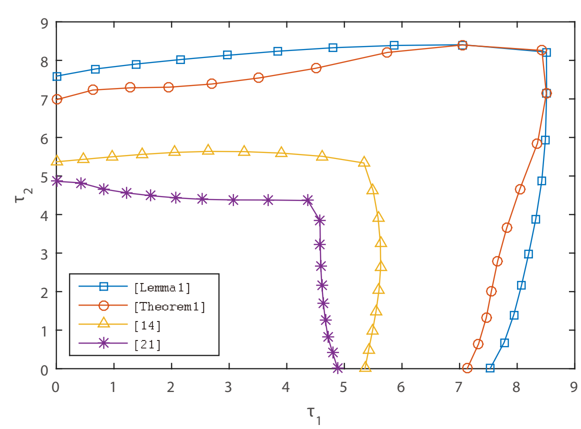

Fig. 3. Stability regions of traditional two-area LFC scheme equipped with PI controllers ($K_P = 0.4, K_I = 0.2$)

Since the delay margins obtained by Theorem 1 and Lemma 1 have the almost same accuracy, and only at some cases,

TABLE II
DELAY MARGINS OF TRADITIONAL TWO-AREA LFC SCHEME WITH PID CONTROLLER ($K_P = 0.2, K_I = 0.2, K_D = 0.2$)

	τ(s)				
θ	[21] (error)	[14] (error)	Lem.1 (error)	The.1 (error)	RealValue
0°	6.47 (23.88%)	6.47 (23.88%)	8.47 (0.35%)	8.46 (0.47%)	8.50
20°	6.89 (23.87%)	7.25 (19.89%)	9.04 (0.11%)	9.04 (0.11%)	9.05
40°	8.45 (23.87%)	8.71 (21.53%)	11.09 (0.09%)	11.09 (0.09%)	11.10
45°	9.13 (23.28%)	9.14 (23.19%)	11.87 (0.25%)	11.87 (0.25%)	11.90
50°	8.64 (21.31%)	8.92 (18.76%)	10.95 (0.27%)	10.95 (0.27%)	10.98
70°	7.04 (21.34%)	7.45 (16.76%)	8.93 (0.22%)	8.93 (0.22%)	8.95
90°	6.62 (21.28%)	6.62 (21.28%)	8.38 (0.36%)	8.38 (0.36%)	8.41

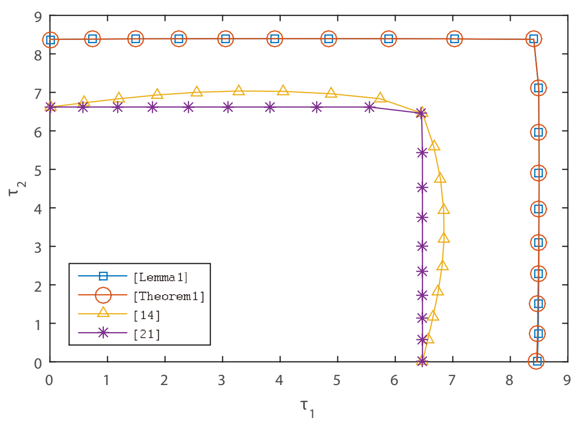

Fig. 4. Stability region of traditional two-area LFC scheme equipped with PID controllers ($K_P = K_I = K_D = 0.2$)

TABLE III
DELAY MARGINS OF DEREGULATED TWO-AREA LFC SCHEME WITH PI CONTROLLER ($K_P = 0.1, K_I = 0.2$)

	τ(s)				
θ	[21] (error)	[14] (error)	Lem.1 (error)	The.1 (error)	RealValue
0°	9.58 (16.70%)	9.75 (15.22%)	11.47 (0.26%)	11.47 (0.26%)	11.50
20°	9.32 (25.62%)	10.43 (16.76%)	12.39 (1.12%)	12.35 (1.44%)	12.53
40°	9.14 (20.38%)	9.95 (13.33%)	**11.45** (0.26%)	11.33 (1.31%)	11.48
45°	9.10 (18.02%)	9.70 (12.61%)	11.07 (0.27%)	11.07 (0.27%)	11.10
50°	9.12 (20.42%)	9.91 (13.53%)	11.44 (0.17%)	11.40 (0.61%)	11.46
70°	9.22 (24.61%)	10.32 (15.62%)	12.12 (0.90%)	12.10 (1.06%)	12.23
90°	9.42 (16.34%)	9.63 (14.48%)	11.23 (0.27%)	11.23 (0.27%)	11.26

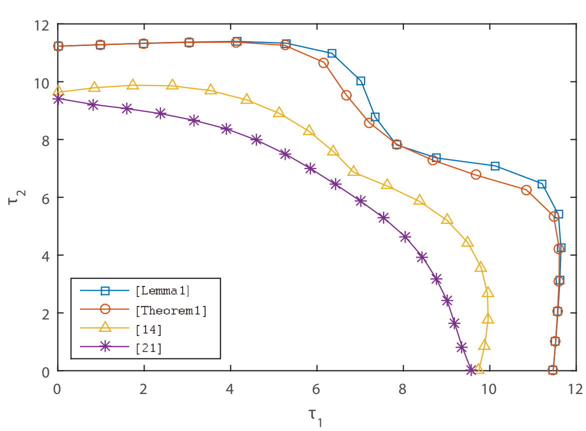

Fig. 5. Stability regions of deregulated two-area LFC scheme with PID controllers ($K_P = 0.1, K_I = 0.2$)

TABLE IV
DELAY MARGINS OF DEREGULATED TWO-AREA LFC SCHEME WITH PID CONTROLLER ($K_P = 0.05, K_I = 0.2, K_D = 0.04$)

	τ(s)				
θ	[21] (error)	[14] (error)	Lem.1 (error)	The.1 (error)	RealValue
0°	9.72 (14.13%)	9.72 (14.13%)	11.30 (0.18%)	11.30 (0.18%)	11.32
20°	8.73 (29.43%)	9.10 (26.43%)	12.31 (0.49%)	12.25 (0.97%)	12.37
40°	8.60 (23.42%)	8.72 (22.35%)	11.21 (0.18%)	11.21 (0.18%)	11.23
45°	8.57 (21.23%)	8.58 (21.14%)	10.85 (0.28%)	10.85 (0.28%)	10.88
50°	8.58 (23.53%)	8.71 (22.37%)	11.19 (0.27%)	11.19 (0.27%)	11.22
70°	8.89 (26.22%)	9.30 (22.82%)	11.88 (1.41%)	11.87 (1.49%)	12.05
90°	9.77 (11.82%)	9.77 (11.82%)	11.06 (0.18%)	11.06 (0.18%)	11.08

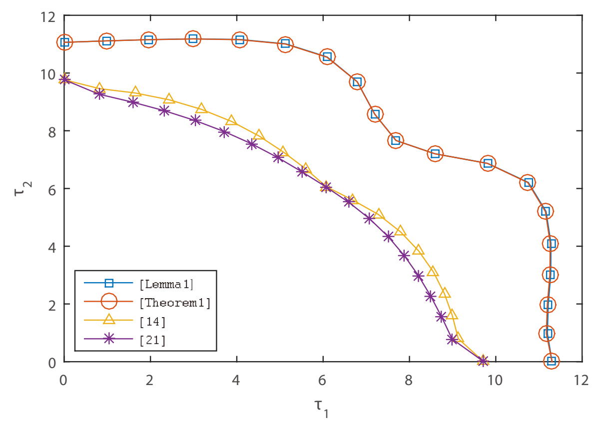

Fig. 6. Stability region of deregulated two-area LFC scheme with PI controllers ($K_P = 0.05, K_I = 0.2, K_D = 0.04$)

Theorem 1's results are little worse than that of Lemma 1. The improvement of calculation efficiency via model reconstruction (will be demonstrated in subsection IV.B) can be obtained with a very small cost of deduction of calculation accuracy.

Note that, Table I and Fig.3 show that for PI case, the calculation error increases when θ tends to move away from the 45 degrees, though the largest error are still much smaller than [14] and [21]. For a fair comparison, same PI gains as [21] and [14] are employed in this paper. To illustrate the relationship between system parameters and the value of calculation errors, more cases with different PI controller gains are calculated and the results are summarized in Table V, which includes two set of small gains and one big gain. It is shown that for smaller PI gains, the errors uniformly vary and all errors approximate to zero, while the case of the big gains ($K_P = 0.4, K_I = 0.3$) shows similar trend as Table I. Although the increase of PI gains leads to larger errors for θ away from 45°, in practice, the value of controller gains is relative small to present a better dynamic performance.

B. Efficiency verification

Several calculation performances of Theorem 1, compared with that of Lemma 1, [14] and [21], are summarized in Table VI and VII. Firstly, the NDVs and the MoLs of these criteria are calculated and listed in Table VI. Then, based on the same calculation environment, i.e., a PC equipped with

TABLE V
DELAY MARGINS OF TRADITIONAL TWO-AREA LFC SCHEME WITH VARIOUS PI CONTROLLERS

	$K_P = 0.1, K_I = 0.1$			$K_P = 0.1, K_I = 0.2$			$K_P = 0.4, K_I = 0.3$		
θ	Lem.1 (error)	The.1 (error)	Real Value	Lem.1 (error)	The.1 (error)	Real Value	Lem.1 (error)	The.1 (error)	Real Value
0°	16.08(0.25%)	16.08(0.25%)	16.12	7.77(0.13%)	7.77(0.13%)	7.78	5.11(7.09%)	4.86(11.64%)	5.5
20°	17.15(0.00%)	17.15(0.00%)	17.15	8.28(0.00%)	8.28(0.00%)	8.28	5.76(1.54%)	5.63(3.76%)	5.85
40°	21.02(0.10%)	21.02(0.10%)	21.04	10.15(0.10%)	10.15(0.10%)	10.16	7.16(0.28%)	7.15(0.42%)	7.18
45°	22.64(0.04%)	22.64(0.04%)	22.65	10.86(0.18%)	10.86(0.18%)	10.88	7.58(0.39%)	7.57(0.53%)	7.61
50°	20.89(0.10%)	20.89(0.10%)	20.91	10.02(0.00%)	10.02(0.00%)	10.02	6.99(0.29%)	6.98(0.43%)	7.01
70°	17.04(0.06%)	17.04(0.06%)	17.05	8.17(0.00%)	8.17(0.00%)	8.17	5.64(1.23%)	5.51(3.50%)	5.71
90°	15.98(0.25%)	15.98(0.25%)	16.02	7.66(0.26%)	7.66(0.26%)	7.68	5.03(6.33%)	4.8(10.61%)	5.37

TABLE VI
VARIABLE COMPUTATIONAL PERFORMANCE OF [14], [21], THEOREM 1 AND LEMMA 1

Controller		[14]			[21]			Lemma 1			Theorem 1		
		$m_{[14]}$	$n_{[14]}$	$t_{[14]}(s)$	$m_{[21]}$	$n_{[21]}$	$t_{[21]}(s)$	m_L	n_L	$t_L(s)$	m_T	n_T	$t_T(s)$
Traditional	PI	27	225	9	23	304	83	45	558	124	29	250	15
	PID	27	225	8	27	468	199	45	558	108	37	388	41
Deregulated	PI	39	455	162	31	514	314	65	1144	1126	33	336	17
	PID	39	455	146	39	962	2232	65	1144	1148	49	676	162

TABLE VII
COMPARISON OF COMPUTATIONAL PERFORMANCE BETWEEN [14], [21], THEOREM 1 AND LEMMA 1

Controller		ratio (%)								
		$m_T/m_{[14]}$	$n_T/n_{[14]}$	$t_T/t_{[14]}$	$m_T/m_{[21]}$	$n_T/n_{[21]}$	$t_T/t_{[21]}$	m_T/m_L	n_T/n_L	t_T/t_L
Traditional	PI	107.4%	111.1%	166.7%	126.1%	82.2%	18.1%	64.44%	44.80%	12.1%
	PID	137.0%	172.4%	512.5%	137.0%	82.9%	20.6%	82.22%	69.53%	38.0%
Deregulated	PI	84.6%	73.8%	10.5%	106.5%	65.4%	5.4%	50.77%	29.37%	1.5%
	PID	125.6%	148.6%	111.0%	125.6%	70.3%	7.3%	75.38%	59.09%	14.1%

an Intel i5 CPU, a 8GB RAM and a 64-bit operation system, and the same presets of calculation procedure, the average calculation time spent on obtaining delay margins for two-area traditional and deregulated LFC schemes with a PI or PID controller is given in Table VI. Meanwhile, the ratio of Theorem 1 to results in [12], [21] and Lemma1 in terms of MoLs, NDVs and time consumed is presented in Table VII.

In order to realize almost accurate delay margins, Lemma 1 is established resorting to the construction of an augmented LKF and the tight estimation of its derivative via a Wirtinger inequality. Hence, compared with the result in [14] via a simpler LKF and more conservative inequality, Lemma 1 possesses the higher NDVs and MoLs, and it takes more time to compute the delay margins.

To make up for Lemma 1's deficiency in time consumption, the reconstructed technique is proposed in this paper, and then, Theorem 1 is obtained. Results in Tables VI and VII show that, in comparison with Lemma 1, the NDVs, the MoLs and the calculation time for Theorem 1 have been reduced greatly, which indicates the calculation efficiency improvement of the stability condition established. Furthermore, for deregulated case, when Theorem 1 and Lemma 1 are compared, the decrease of the NDVs, the MoLs and the computing time is more significant, which implies the model reconstruction technique proposed in this paper makes increased contributions for the determination of delay margins in higher-dimensional systems.

Although, in some cases, Theorem 1 still consumes more time for determining delay margins than [14], it can be acceptable and reasonable since Theorem 1 can realize almost accurate results and there is a trade-off between accuracy and computation efficiency. Moreover, according to Table VII, under deregulated environment, the distinction between Theorem 1 and [14] is shortened in terms of MoLs, NDVs and time demanded. It can be expected that, for higher-dimensional system, the superiority of Theorem 1 will be more obvious.

As for the results in [21], although they have lower MoLs, they own more NDVs and need more time to compute delay margins than Theorem 1 does, especially in deregulated case. It should be noted that, since the method presented in [21] is too strict in decomposing system model to be used for PID cases, from Table VI, the rapidly increased amount of time is required when the delay margin is calculated.

V. CONCLUSION

This paper has investigated the delay-dependent stability of traditional and deregulated large-scale multi-area LFC schemes in order to simultaneously improve the calculation accuracy and efficiency of delay margins. Via exploiting the sparse feature of the LFC model and using a transition matrix, the reconstructed technique has been proposed to equivalently represent the original model with the delay-free part and delay-related part. A novel augmented Lyapunov functional has been constructed mainly based on the delay-related states, and its derivative has been estimated via the Wirtinger inequality so as to reduce the conservatism of the developed criteria.

Case studies have been carried out based on two-area traditional and deregulated LFC schemes. Results show that the proposed stability criterion which is one type of Lyapunov stability based indirect methods, can almost achieve the accurate delay margin obtained by the frequency domain method. Moreover, the calculation efficiency has also been improved via the model reconstruction techniques, with a very small cost of reduction of accuracy, comparing with the stability criterion obtained based on the model before reconstruction.

The proposed criteria will be further tested in large-scale multi-area LFC schemes. Moreover, similar model reconstruction techniques could be applied to other time delay power systems such as wide-area damping controller [26], for a better exploitation of the special characteristics of time delay power system model during the formulation of stability criteria.

APPENDIX I

Table VIII shows the parameters used in the traditional two-area LFC scheme with one Genco and Disco included in each area. The parameters of a deregulated two-area LFC scheme with two Gencos and two Discos in each area are given in Table IX.

TABLE VIII
TRADITIONAL TWO-AREA LFC SCHEME

Parameters	T_t	T_g	R	D	β	M	α	T_{12}
Area1	0.30	0.10	0.05	1.00	21.0	10	1.00	0.1968
Area2	0.40	0.17	0.05	1.50	21.5	12	1.00	

TABLE IX
DEREGULATED TWO-AREA LFC SCHEME

Parameters	($k-i$: k in area i)					Areas	
	1-1	2-1	1-2	2-2		1	2
T_t	0.32	0.30	0.30	0.32	M	0.1667	0.2084
T_g	0.06	0.08	0.06	0.07	D	0.0084	0.0084
R	2.4	2.5	2.5	2.7	β	0.4250	0.3966
α	0.5	0.5	0.5	0.5	T_{12}	0.2450	

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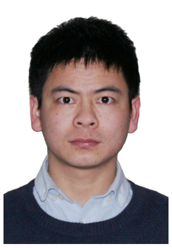

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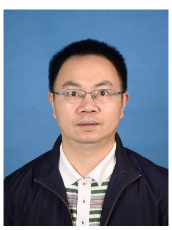

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